THEORY OF PENETRATION BY JETS OF NON-LINEAR VELOCITY AND IN LAYERED TARGETS

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Based on the Pugh-Eichelberger theory, two sets of penetration equations have been developed. The first one involves the penetration of a monolithic target by a shaped charge jet with non-linear velocity distribution. The second involves the penetration by shaped charge jet on layered targets.

Introduction

The classical penetration theory by shaped charge jet into semi-infinite target is based on a constant velocity jet. It is applicable to a small segment of the jet where its velocity can be considered constant. When applied to acutual jets, it is customary to consider the jet velocity varies linearly along its length, which is true for most jets from conical liners.

Recent research by U.S. Air Force has indicated that most jets from modern shaped charges have a non-linear velocity distribution, Ref [1]. In this paper, equations are derived for these non-linear velocity jets. The non-linear velocity distribution is approximated by two linear segments. Each of these segment is associated with a virtual origin.

With the same jet tip and jet tail velocities, the one with a concave downward velocity distribution has deeper penetration than the linear (straight line) case.

Formulas are also derived for jet penetration into layered target. It includes spaced armor, where the air space is considered as a layer with negligible density. It can also be generalized to targets of continuous but varying density distribution. It is interesting to note that for a given set of layered plates with different density, less penetration can be made by a given jet, if the denser layer is arranged near the surface, closer to the charge.

Penetration by Linear-velocity jets

We shall start with Pugh-Eichelberger's hydrodynamic penetration theory without strength effect. Let v, u, ρ_j, ρ_t be the jet velocity, penetration velocity, jet density, and target density, respectively. From Bernoulli's equation

$$\rho_j (v-u)^2 = \rho_t u^2$$

we obtain

$$u = \frac{v}{1+\gamma}, \text{ where}$$

$$\gamma = (\rho_t/\rho_j)^{1/2}$$

The jet velocity v is not constant along the jet, but varies with x, a parameter identifying the jet particle.

The penetration curve (bottom of the penetration hole), has the equation

$$\frac{d\zeta}{dt} = u \qquad (1)$$

or

$$\frac{d\zeta}{dt} = \frac{v(x)}{1+\gamma} .$$

where ζ is the space variable. Eq (1) is the differential equation of the penetration curve. Assuming the existance of a virtual origin (V.O.), Ref. [3], the particle path lines have the equation:

$$\zeta = v(x)t \qquad (2)$$

Combining (1) and (2), and eliminating v(x), we have

$$\frac{d\zeta}{dt} = \frac{\zeta}{(1+\gamma)\, t} \qquad (3)$$

Integrating (3) gives

$$\zeta = k\, t^{1/(1+\gamma)}$$

where k is a constant of integration.

Let t_1 be the time the jet tip reaches the target surface, located at ζ=s, therefore,

$$s = kt_1^{\,1/(1+\gamma)}$$

or

$$k = st_1^{\,-1/(1+\gamma)}$$

The equation in the t,ζ plane for the penetration curve is then

$$\zeta = s\left(\frac{t}{t_1}\right)^{1/(1+\gamma)}$$

or

$$\zeta^{(1+\gamma)} = s^{(1+\gamma)}\left(\frac{t}{t_1}\right) \qquad (4)$$

Substituting (2) into (4) and noting that $s = v_1 t_1$, where v_1 is the jet tip velocity, we have

$$\zeta^{(1+\gamma)} = s^{(1+\gamma)}\left(\frac{\zeta}{v}\frac{v_1}{s}\right)$$

or

$$\zeta = s\left(\frac{v_1}{v}\right)^{1/2} \qquad (5)$$

The penetration depth P by the jet particle with velocity v is then

$$P = \zeta - s$$

or

$$P = s\left[\left(\frac{v_1}{v}\right)^{1/\gamma} -1\right] \qquad (6)$$

Figure 1 is a schematic showing the particle path lines and the penetration curve in the space-time plane. Note that s is the standoff, measured from the V.O. to the surface of the target, v_1 is the tip velocity, v is the velocity of the particle that gives the depth P. As a check, we may find the slope of the penetration curve by taking d/dt of (4) after, some simplication, it gives

$$\frac{d\zeta}{dt} = \frac{1}{1+\gamma} v(x). \qquad (7)$$

which agrees with Eq. (3).

Penetration by Non-linear Velocity Jets

Next, let us consider the case where the velocity distribution is not linear, which implies that a single virtual origin does not exist. For simplicity, we shall use a bi-linear velocity distribution to represent the non-linear case. As shown in Figure 2, the velocity distribution is given by two straight line segments, AD and DB. This will be compared with the linear velocity distribution ACB. Note that points C and D have the same velocity V_2.

For the linear case ACB, there is a virtual origin, which will be taken as the origin of our t,ζ coordinates.

The penetration P of the linear case, as given by Eq. (6), is

$$P = s\left[\left(\frac{v_1}{v_3}\right)^{1/\gamma} - 1\right] \qquad (8)$$

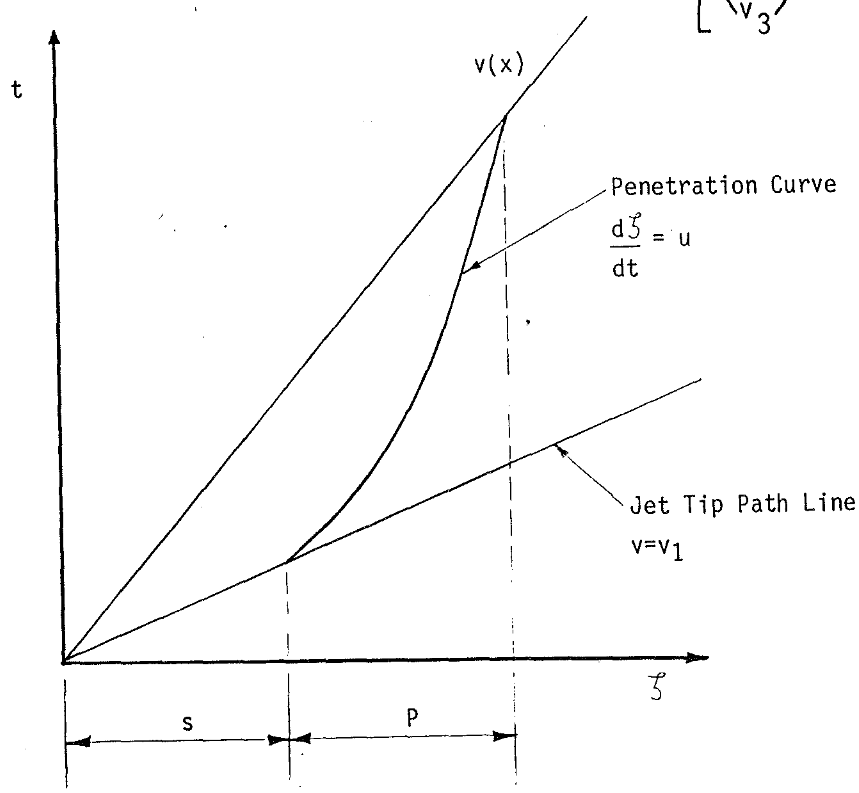

Figure 1. The Jet Particle Path Lines and the Penetration Curve in the Space-time Plane.

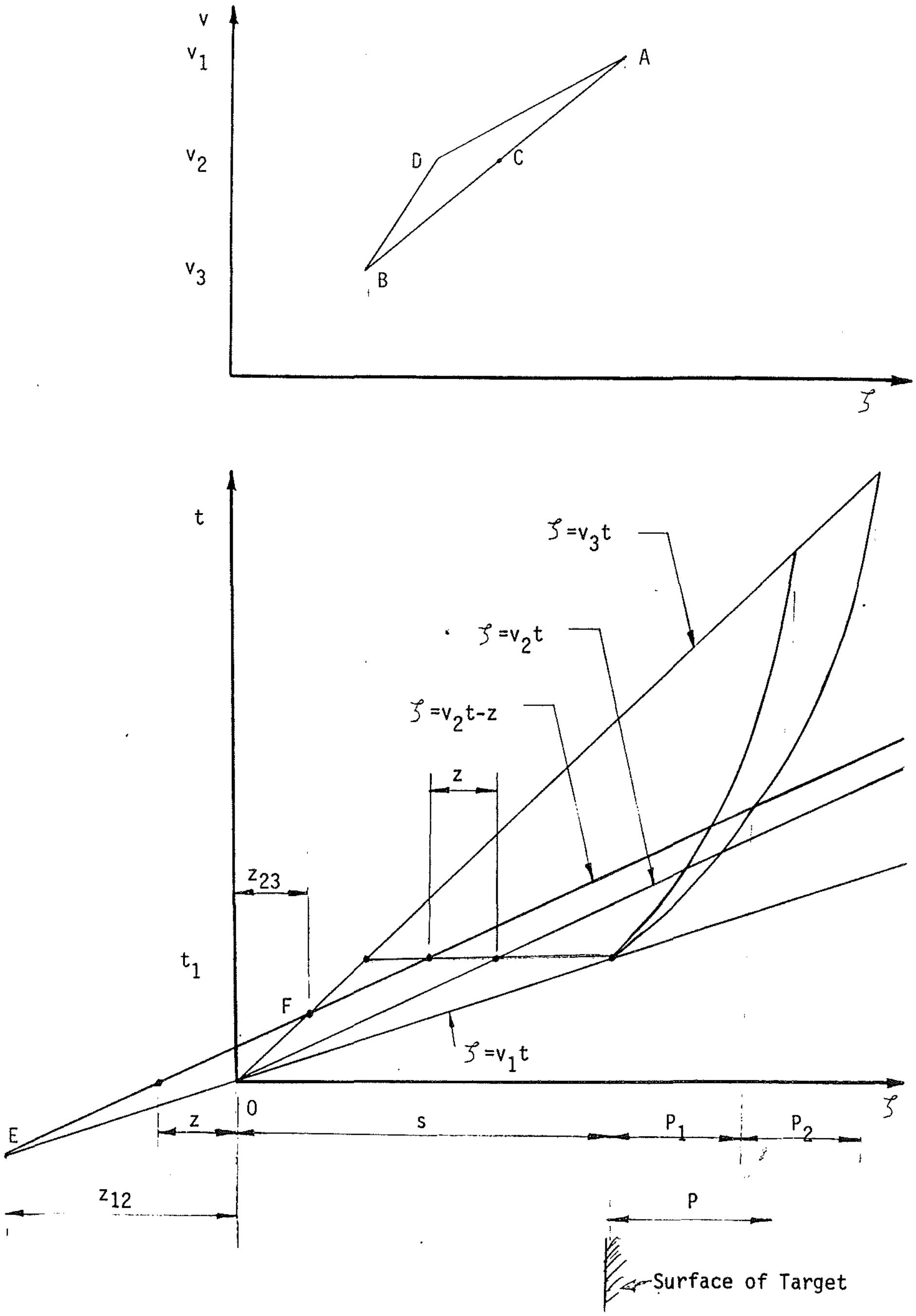

Figure 2. Velocity-Space and Time-Space Diagram of a Jet with Bi-linear Velocity Distribution and their Penetration Curves.

where v_1 and v_3 are the jet tip and tail velocities, respectively.

Now let us consider the nonlinear case ADB. Let the distance CD be z. It can be shown, Ref. [2], that the particle path line corresponding to D is

$$\zeta = v_2 t - z \qquad (9)$$

where v_2 is the velocity of both points C and D. In the t-ζ plane, the path line of point D is the straight line with a horizontal intercept (-z). Solving the two straight line equations $\zeta = v_1 t$ and $\zeta = v_2 t - z$, we find the intersection of these two lines, point E in Figure 2, with the horizontal coordinate

$$z_{12} = \frac{v_1 z}{v_2 - v_1} \qquad (10)$$

Similarly, the intersection between $\zeta = v_3 t$ and $\zeta = v_2 t - z$, point F, has the coordinate

$$z_{23} = \frac{z\, v_3}{v_2 - v_3} \qquad (11)$$

The penetration due to the segment AD can be calculated from Eq. (6) by considering E as the virtual origin, and the standoff as $s - z_{12}$, or,

$$P_1 = \left(s - z_{12}\right)\left[\left(\frac{v_1}{v_2}\right) - 1\right] \qquad (12)$$

$$= \left(s - \frac{v_1 z}{v_2 - v_1}\right)\left[\left(\frac{v_1}{v_2}\right)^{1/\gamma} - 1\right].$$

The penetration due to the jet segment DB may be calculated by using the virtual original at point F, or

$$P_2 = \left(s - z_{23} + P_1\right)\left[\left(\frac{v_2}{v_3}\right)^{1/\gamma} - 1\right]. \qquad (13)$$

The total penetration P_t for this bi-linear case is $P_1 + P_2$, and after some algebraic simplification, it reduces to

$$P_t = P_1 + P_2$$

$$= s\left(\frac{v_1'}{v_3'} - 1\right) + z\left[\frac{v_1}{v_3'}\left(\frac{v_1' - v_2'}{v_3 - v_2}\right) - \frac{v_3}{v_3'}\left(\frac{v_2' - v_3'}{v_2 - v_3}\right)\right] \qquad (14)$$

where v_1' is used for $v_1^{1/\gamma}$

For a copper jet penetrating a steel target, the density ratio γ is close to unity. For simplicity, consider the case of $\gamma=1$, then $v_1' = v_1$, and

$$P_t = (s+z)\left(\frac{v_1}{v_3} - 1\right) \qquad (15)$$

This penetration is larger than the linear velocity case. It is equivalent to increasing the stand-off distance from s to s+z. The ratio of

P_t/P is

$$\frac{P_t}{P} = 1 + \frac{z}{s} \qquad (16)$$

This shows that the nonlinear velocity case with $z > 0$ has more penetration than the linear case. The difference is z/s percent more, where z is the ζ intercept of the particle path line D, or the distance CD in Figure 2. This may also be seen by simple reasoning as follows. The particle with velocity V_2, at point D, is "formed" earlier than the particle with the same velocity V_2, point C, in the linear case. If the tip and tail (A and B) are the same between the linear and nonlinear cases, then all particles in the nonlinear case must have their point of formation, or equivalent virtual origins to the left of the point 0. Their effective standoff must be larger than s, therefore more penetration.

The same argument and results can be applied to the case where $z < 0$, or a concase upward velocity distribution. When $z < 0$, the penetration is smaller than the linear case. In general, the velocity distribution curve, if not linear, will give more penetration when concave downward.

The net difference in penetration due to nonlinear velocity is of minor effect at large standoff, since when s is large, z/s is small for a given z. Figure 3 shows the velocity of the jet from a bi-conic charge Ref. [1]. The dot-points are the velocity of different mass particles in the jet as determined by flash x-ray pictures and a mass-summing technique. The spacial distribution of the velocity is non-linear; it can be approximated by two linear segments, AD and DB, as shown in the figure. The dotted line AB connecting the tip and tail velocities is drawn in for comparison. The distance DC, which is z in Eqs. (14) and (15), is about 125 mm, (This distance remains constant as the jet is moving forward, because the velocities at D and C are identical). At a short stand-off of 125 mm, this non-linear jet can penetrate approximately twice as deep as a

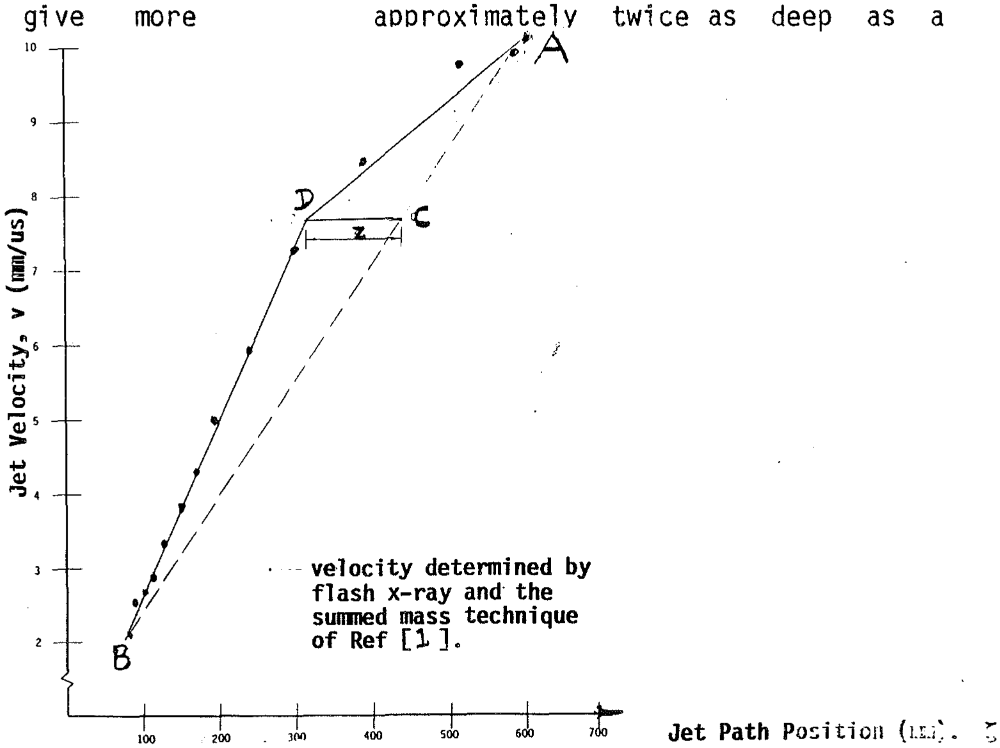

Figure 3. Jet Velocity Distribution of a Biconic-liner Charge, Taken at 100 usec After Detonation.

linear jet with the same tip and tail velocities.

The penetration of a shaped charge jet with non-linear concave downward velocity distribution in semi-infinite target is z/s percent more than that from a corresponding charge with identical tip and tail speeds, but with linear velocity distribution. Here, s is the distance between the surface of the target and the virtual origin of the linear case, or the intersection of the particle path lines formed by the tip and tail; z is maximum distance between the position of a particle and the position of a particle with the same velocity, but with linear velocity distribution with respect to the tip and tail particles. Also, z is positive when the velocity distribution curve is concave downward.

Penetration of Layered Target

The incompressible hyrodynamic theory of jet penetration will be extended to the case of layered target. The penetration as a function of jet velocity, Eq. (6), can be inverted to give

$$v = v_0 \left(1 + \frac{P}{s}\right)^{-\gamma} \qquad (17)$$

This equation gives the particle velocity in a jet that will penetrate a depth P. Since the jet velocity of a given jet particle is essentially constant, it is convenient to use jet velocity to identify the jet particle.

Consider now a layered target as shown in Figure 4, where:

d_i = thickness of the ith layer

s_i = standoff distance from V.O. to the front of the ith layer.

$\gamma_1 = (\rho_{t1}/\rho_j)^{1/2}$

v_0 = jet tip velocity

v_1 = velocity of jet particle penetrating the back surface of the first layer

ρ_{ti} = density of the first target layer

Then

$$v_1 = v_0 \left(1 + \frac{d_1}{s_1}\right)^{-\gamma_1}$$

or,

$$v_1 = v_0 \left(\frac{s_1}{s_2}\right)^{\gamma_1} \qquad (18)$$

The velocity emerging from the back of the second target layer is then,

$$v_2 = v_1 \left(\frac{s_2}{s_3}\right)^{\gamma_2}$$

$$= v_0 \left(\frac{s_1}{s_2}\right)^{\gamma_1} \left(\frac{s_2}{s_3}\right)^{\gamma_2}$$

This may be extended to a target of ith layer as

$$v_i = v_0 \left(\frac{s_1}{s_2}\right)^{\gamma 1} \left(\frac{s_2}{s_3}\right)^{\gamma 2} \text{---} \left(\frac{s_i}{s_{i+1}}\right)^{\gamma i}$$

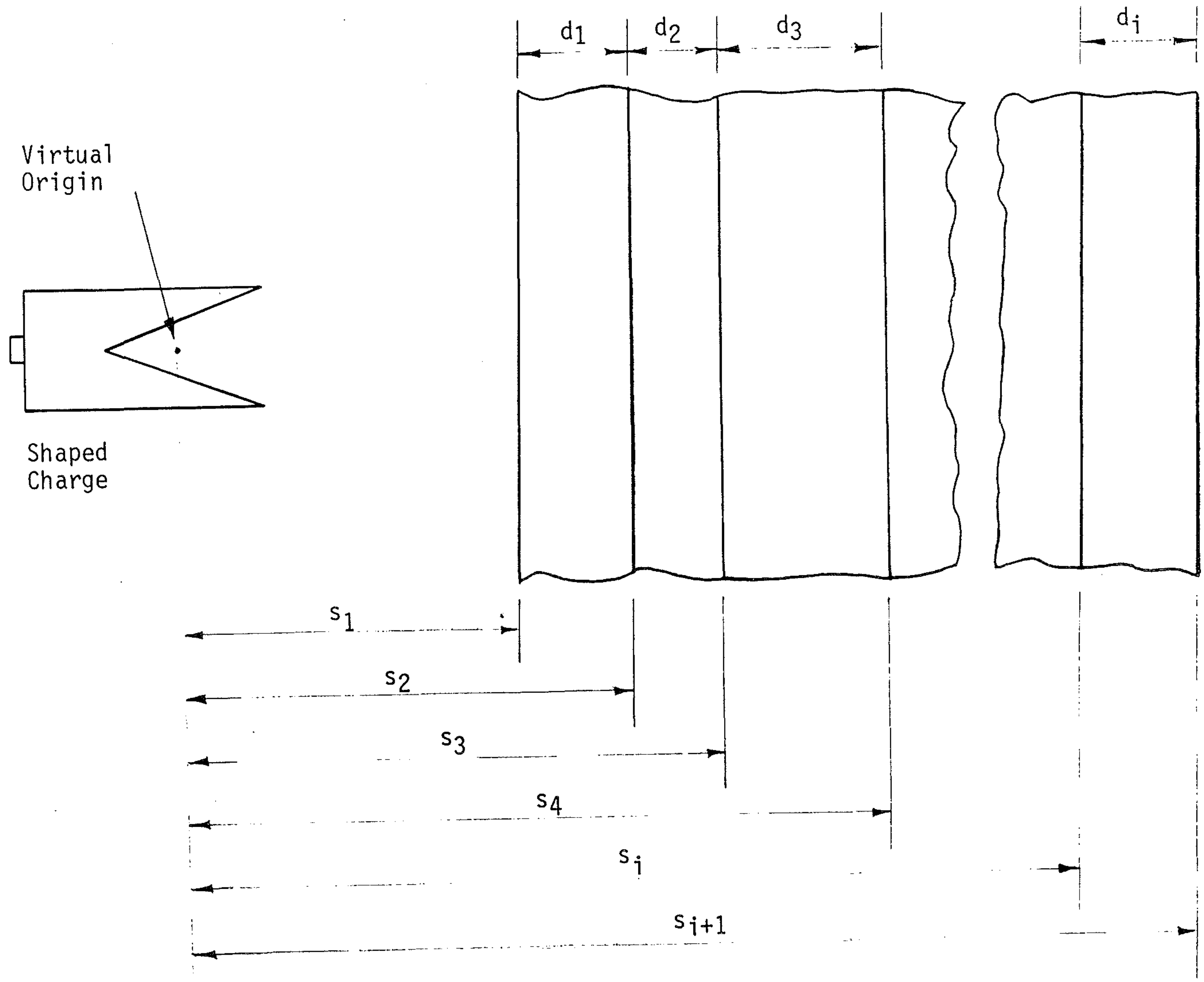

Figure 4. Geometry of a Shaped Charge with Virtual Origin Penetrating a Layered Target.

$$= v_0 \prod_{k=1}^{i} \left(\frac{s_k}{s_{k+1}} \right)^{\gamma_1} \qquad (19)$$

In this equation, v_i is the jet particle velocity that will emerge from the back of the ith target layer. When the target thickness is not too large and the actual jet can penetrate through all layers, Eq. (19) is applicable. When the target is too thick for the jet to penetrate through, the penetration will stop at a jet velocity v_{min}, as discussed in Ref. [4]. The jet mass with velocity below v_{min} will have no penetration ability.

In deriving Eq. (19), the target strength and the jet break-up have both been ignored; it is applicable at short stand-off where the jet has not broken and its velocity is high. It gives a measure of the amount of jet, identified by the jet particle velocity, that will be "consumed" by a given layered target.

As a numerical example, consider the case of a copper jet perforating a target of two 50-mm layers, one with the RHA in front and aluminum in the back, the other just the opposite, as shown in Figure 5. A copper jet with tip a velocity of 10 km/sec is assumed. The front of the layered plates are placed at a stand-off distance of 75 mm from the charge. With the density of copper, steel, and aluminum of values 8.96, 7.87, and 2.7 gr/cc., respectively, the corresponding values of γ for steel (RHA) is 0.937, and for aluminum 0.549. The emerging jet velocities calculated from Eq. (19) are 0.515 and 0.551 km/sec, respectively. If the minimum jet velocity for penetration is taken as 2 km/sec, the difference of the residual jet length is more than 10%.

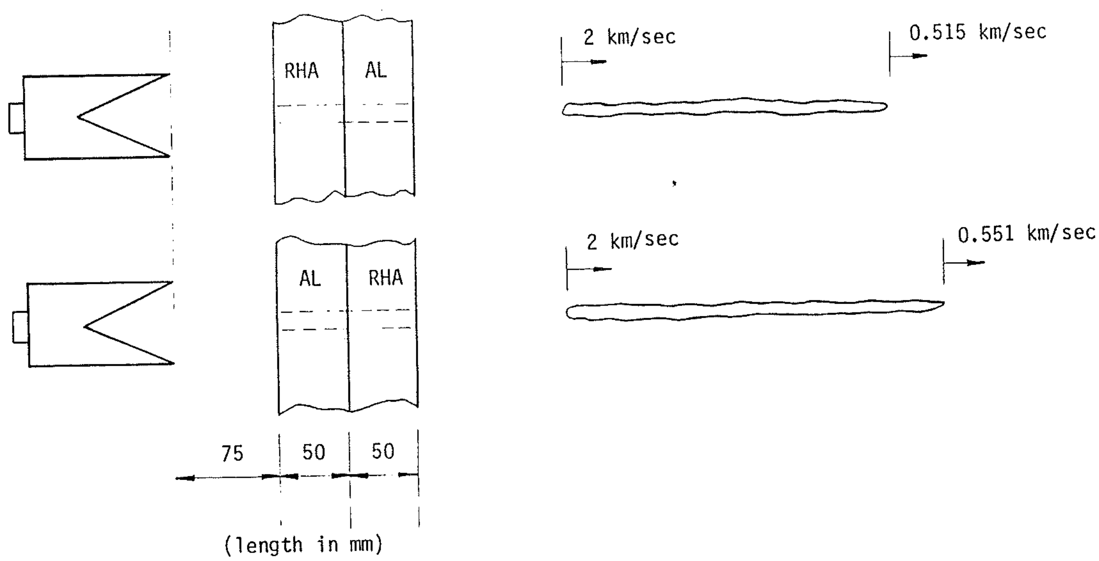

Figure 5. Schematic Showing Residual Jet Length and Velocity of a Copper Jet with Initial Tip Velocity of 10 km/sec after Perforating an RHA-Aluminum, and an Aluminum-RHA Plates.

The shaped charge designer should not allow any material, such as seekers, to be placed close to the charge. When present, the high density material should be farther away from the charge.

Conclusions

Based on the hydrodynamic penetration theory, it is shown that with the same tip and tail velocities, the jet with a non-linear concave downward velocity distribution gives deeper penetration than the linear one. Most modern shaped charges do have a concave downward distribution, even though most were developed from trial and error experimentation.

A simple formula for jet penetration in layered-target is derived. It depends on each layer's density, standoff distance from the charge, and its thickness. High density layers consume more jet length, and should be arranged away from the charge, especially at short standoff from the warhead design point of view.

References

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