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GURNEY FORMULAS FOR EXPLOSIVE CHARGES SURROUNDING RIGID CORES

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The Gurney problem of the acceleration of liners by explosive charges having rigid cores is rigorously examined, and new Gurney formulas are derived. First, it is shown that the usual assumptions of linear velocity distribution and uniform density throughout the explosive products are not equivalent for cylindrical and spherical charges when a rigid core is present. This is done by deriving the density distribution consistent with a linear velocity distribution, and the velocity distribution for a uniform density. Then, each of these consistent sets of conditions is used to derive a new Gurney formula. This paper also compares with, and clarifies the differences among, several previous Gurney formulas for such charges. Comparisons with hydrocode computations are included.

INTRODUCTION

The well-known Gurney model (Gurney, 1943) is based on two assumptions about the explosive's behavior during acceleration of the metal: linear distribution of velocity and uniform density. Each assumption is used in a different part of the model, which is based on the principle of conservation of energy. Specifically, the change in the explosive's internal energy is equated to the kinetic energy of the liner and explosive products gases; thus,

$$C E_0 - \int_{gas} E\, dC = \frac{1}{2} M V_0^2 + \frac{1}{2} \int_{gas} V^2 dC \qquad (1)$$

where C and M are the masses of the charge and metal liner; E is the explosive's internal energy, with initial value E_0; V_0 is the velocity of the liner; and V is the particle velocity within the explosive products. If, as Gurney suggested, the explosive products expand adiabatically according to the ideal-gas law, then

$$E = E_0 \left(\frac{\rho}{\rho_0}\right)^{\gamma - 1} \qquad (2)$$

where ρ and ρ_0 are the current and initial explosive densities. The assumption of uniform density simplifies integration of the current internal energy over the explosive, and the assumption of a linear particle-velocity distribution simplifies that of the explosive's kinetic energy. However, as shall be shown, for a cylindrical or spherical charge surrounding a rigid central core, these two assumptions are not consistent. This raises the question of how to derive a rigorously self-consistent Gurney formula for such charges.

In this paper, this question is resolved by deriving, first, the non-linear velocity distribution that arises from the assumption of uniform density and, second, the non-uniform density distribution that arises from the assumption of linear velocity. Then each of these consistent sets of conditions is used to derive a self-consistent formula for the liner's velocity. These formulas are compared with previous Gurney formulas for such charges and the results of hydrocode computations.

ANALYSIS OF THE GURNEY ASSUMPTIONS

In this section, the usual Gurney assumptions of uniform density and linear velocity distribution in the explosive are examined. It is shown that these assumptions are equivalent and consistent except for cylindrical and spherical charges surrounding rigid cores.

Uniform density. The velocity distribution in an explosive about a rigid core under the assumption that the explosive gas's density remains always uniform was first addressed by Sterne (1951) for spherical charges. Here, his analysis is generalized to include planar (n = 1), cylindrical (n = 2), and spherical (n = 3) charges, and the resulting non-linear explosive velocity distribution is expressed in terms of both Lagrangian and Eulerian spatial coordinates.

Let R denote the Lagrangian radial coordinate (initial position) of any explosive particle, and r denote its Eulerian radial coordinate (current position), as shown in Fig. 1. Now, let μ denote the fraction of explosive mass between the core and a given particle, i.e., within a radius r or R; thus, μ is a Lagrangian mass coordinate. Then,

$$\mu(R) = \frac{R^n - R_c^n}{R_0^n - R_c^n} \; ; \; \mu(r) = \frac{r^n - R_c^n}{r_0^n - R_c^n} \tag{3}$$

where R_0 and r_0 are the initial and current outer radii of the explosive and R_c is the radius of the rigid core. Solving for r yields

$$r = \left[(r_0^n - R_c^n)\,\mu + R_c^n\right]^{1/n}$$

Since μ is a Lagrangian coordinate, the velocity of any particle is $\dot{r} = (\partial r / \partial t)_{\mu = constant}$, which yields

$$\dot{r} = \mu V_0 \left[\left(1 - \frac{R_c^n}{r_0^n}\right) \mu + \frac{R_c^n}{r_0^n}\right]^{-(n-1)/n}$$

where $V_0 = \partial r_0/\partial t$ is the current liner velocity. Substituting for μ according to Eq. (3) yields the velocity distributions over the initial position R,

$$V(R) = V_0 \, \frac{R^n - R_c^n}{R_0^n - R_c^n} \left[\left(1 - \frac{R_c^n}{r_0^n}\right)\left(\frac{R^n - R_c^n}{R_0^n - R_c^n}\right) + \frac{R_c^n}{r_0^n}\right]^{-(n-1)/n} \tag{4}$$

and over the current position r,

$$V(r) = V_0 \frac{r^n - R_c^n}{r_0^n - R_c^n} \left[\left(1 - \frac{R_c^n}{r_0^n}\right)\left(\frac{r^n - R_c^n}{r_0^n - R_c^n}\right) + \frac{R_c^n}{r_0^n}\right]^{-(n-1)/n} \tag{5}$$

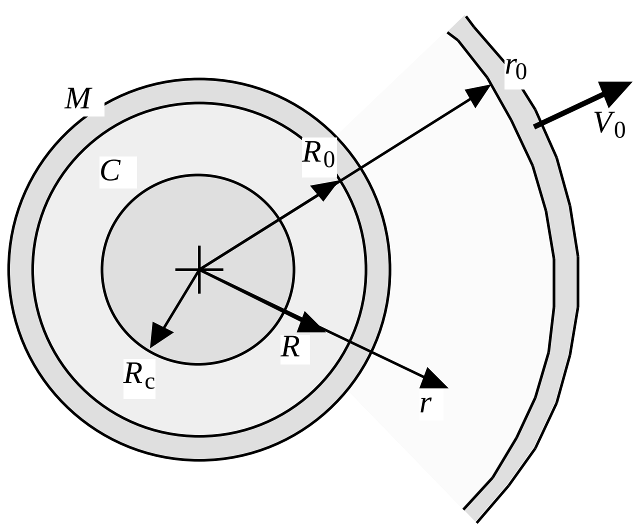

Fig. 1. Explosive charge surrounding a rigid core.

Both expressions are non-linear except for two cases: for planar charges (n = 1), for which they reduce to $V(R) = V_0 \frac{R - R_c}{R_0 - R_c}$; $V(r) = V_0 \frac{r - R_c}{r_0 - R_c}$; and for all charges having no core, $R_c = 0$, for which $V(R) = V_0 \frac{R}{R_0}$; $V(r) = V_0 \frac{r}{r_0}$, all of which are linear. Thus, the assumption of uniform density implies a linear velocity except for cylindrical and spherical charges surrounding finite cores.

Linear velocity distribution. We now examine the assumption that the velocity distribution in the explosive products is always linear in the initial coordinate R. First, this condition is shown to be equivalent to linearity also in the Eulerian coordinate r. The position r of the particle originally at R is given at any time t by

$$r = R + \int_0^t V(R, t)\, dt$$

If V is always linear in R,

$$V(R, t) = V_0(t)\, \frac{R - R_c}{R_0 - R_c} \tag{6}$$

then

$$r = R + \frac{R - R_c}{R_0 - R_c} \int_0^t V_0(t)\, dt = R + \frac{R - R_c}{R_0 - R_c} \left[r_0(t) - R_0 \right] \tag{7}$$

which says that r is linear in R. Thus, V must be linear also in r, as we intended to show.

We now derive the density distribution based on this assumption. Consider an initial interval dR of explosive at density ρ_0. The explosive products originating here will later occupy an interval dr at density ρ. By conservation of mass, $\rho_0\, R^{n-1}\, dR = \rho\, r^{n-1}\, dr$. Solving for ρ yields $\rho = \rho_0\, (R / r)^{n-1}\, (dr / dR)^{-1}$, which, by Eq. (7), becomes

$$\rho(r) = \rho_0\, \frac{R_0 - R_c}{r_0 - R_c} \left[\frac{R_0 - R_c}{r_0 - R_c} + \frac{R_c}{r} \left(1 - \frac{R_0 - R_c}{r_0 - R_c} \right) \right]^{n-1} \tag{8}$$

At any time after onset of motion ($r_0 > R_0$), this varies with r (and thus also with R), except for planar charges (n = 1) or the case of no core ($R_c = 0$). As expected, the density ρ decreases with r.

SIMPLE MODEL OF CASING RUPTURE

As the metal liner is driven outward by the explosive, it stretches in the circumferential and (to a much lesser extent) axial directions. Eventually, the liner fractures, and the explosive products gases leak out between the fragments, reducing their pressure. A simple model to account for this was suggested by Thomas (1944) and adopted by several others, including Sterne (1947, 1951) and Henry (1967). It is based on the assumption that, once the liner expands to a certain multiple of its initial radius, it is no longer accelerated. Thus, any internal energy remaining in the explosive products does not contribute to the liner's final velocity.

CONSISTENT FORMULAS FOR CORED CHARGES

In this section, formulas for the liner velocity are derived based on rigorous application of each of the above assumptions.

Uniform density. By using the condition of uniform density with the proper non-linear velocity distribution, Eq. (4), a consistent Gurney formula may be derived. If the density at each instant is uniform, then $\rho = \rho_0\, (R_0^n - R_c^n)/(r_0^n - R_c^n)$, and the total internal energy of the explosive is simply

$$\int_{gas} E\,dC = \int_{gas} E_0 \left(\frac{R_0^n - R_c^n}{r_0^n - R_c^n}\right)^{\gamma - 1} dC = C\,E_0 \left(\frac{R_0^n - R_c^n}{r_0^n - R_c^n}\right)^{\gamma - 1}$$

The non-linearity of Eq. (4) complicates the integral of the explosive's kinetic energy. The resulting Gurney formula for cylindrical charges is

$$V_0 = \sqrt{2E_0}\left\{\frac{M}{C} + \frac{1}{2}\,\frac{1 - 4b^2 + 3b^4 - 8b^4 \ln b}{(1 - b^2)^3}\right\}^{-1/2} \sqrt{1 - \left(\frac{R_0^2 - R_c^2}{r_b^2 - R_c^2}\right)^{\gamma - 1}} \tag{9}$$

where $b = R_c/r_b$, in which r_b is the value of the radius r_0 at which the expanding casing bursts and acceleration of the liner ceases. The corresponding formula for spherical charges is

$$V_0 = \sqrt{2E_0}\left\{\frac{M}{C} + \frac{3}{5}\,\frac{1 + 3b + 6b^2 + 5b^3}{(1 + b + b^2)^3}\right\}^{-1/2} \sqrt{1 - \left(\frac{R_0^3 - R_c^3}{r_b^3 - R_c^3}\right)^{\gamma - 1}} \tag{10}$$

If bursting is neglected ($r_b \to \infty$), then each of these formulas approaches the corresponding classical Gurney formula for no core, summarized by Thomas (1944) as

$$V_0 = \sqrt{2E_0}\left[\frac{M}{C} + \frac{n}{n + 2}\right]^{-1/2} \tag{11}$$

Linear velocity distribution. By using the assumption of linear velocity distribution, Eq. (6), with the associated distribution of density, a consistent Gurney formula may be derived. The non-uniform density distribution, Eq. (8), complicates the integral of the explosive's internal energy. For cylindrical charges, the resulting formula is

$$V_0 = \sqrt{2E_0}\sqrt{1 - f_2\left(\frac{R_c}{R_0}, \frac{r_0}{R_0}, \gamma\right)}\left\{\frac{M}{C} + \frac{1}{6}\,\frac{3R_0 + R_c}{R_0 + R_c}\right\}^{-1/2} \tag{12}$$

where f_2 is a complicated function that represents the fraction of energy remaining in the explosive when the casing bursts. The formula for spherical charges is

$$V_0 = \sqrt{2E_0}\sqrt{1 - f_3\left(\frac{R_c}{R_0}, \frac{r_0}{R_0}, \gamma\right)}\left\{\frac{M}{C} + \frac{1}{10}\,\frac{6R_0^2 + 3R_0R_c + R_c^2}{R_0^2 + R_0R_c + R_c^2}\right\}^{-1/2} \tag{13}$$

The general expression for the function f_n in these equations is

$$f_n(a, \psi, \gamma) = \frac{n}{1 - a^n}\left(\frac{1 - a}{\psi - a}\right)^{\gamma} \int_0^{\psi - a} \left(a + \frac{1 - a}{\psi - a}\,s\right)^{\gamma(n - 1)} (a + s)^{(1 - \gamma)(n - 1)}\,ds \tag{14}$$

where $a = R_c/R_0$ and $\psi = r_0/R_0$. This seems integrable in closed form only for integral values of γ, for which the value $\gamma = 3$ is a good approximation for condensed explosives; the resulting expressions are, for cylindrical charges ($n = 2$),

$$f_2(a, \psi, \gamma = 3) = \frac{2}{1 - a^2}\left(\frac{1 - a}{\psi - a}\right)^5 \left\{(1 - a)\frac{\psi + a}{2} + 3a(\psi - 1) + a^2\left(\frac{\psi - 1}{1 - a}\right)^2\left[3\,\frac{1 - a}{\psi - a}\ln\left(\frac{\psi}{a}\right) + 1 - \frac{1}{\psi}\right]\right\}$$

and for spherical charges ($n = 3$),

$$f_3(a, \psi, \gamma = 3) = \frac{3}{1 - a^3}\left(\frac{1-a}{\psi - a}\right)^3 \left\{ \begin{aligned} &\frac{1}{3}\left(\frac{1-a}{\psi-a}\right)^6(\psi^3 - a^3) + 3\left(\frac{1-a}{\psi-a}\right)^5 a(\psi - 1)\left[\psi + a + 5a\frac{\psi-1}{1-a}\right] + \\ &+ a^3\left(\frac{1-a}{\psi-a}\right)^2\left(\frac{\psi-1}{\psi-a}\right)^3\left[20\frac{1-a}{\psi-a}\ln\left(\frac{\psi}{a}\right) + 15\frac{\psi-1}{\psi}\right] + \\ &+ \frac{1}{3}\left(\frac{\psi-1}{\psi-a}\right)^6\frac{a^3}{\psi^3}\left[8\psi(\psi+a)(1-a) + \psi^3 - a^2\right] \end{aligned} \right\}$$

COMPARISONS WITH PREVIOUS FORMULAS

Several Gurney formulas for such charges have been previously derived. Sterne (1951) applied the assumption of uniform density to the spherical case, but only in the explosive's kinetic energy, deriving the formula

$$V_0 = \sqrt{2E_0}\left[\frac{M}{C} + \frac{3}{5}\frac{1 + 3b + 6b^2 + 5b^3}{(1 + b + b^2)^3}\right]^{-1/2} \tag{15}$$

However, his earlier analysis of the cylindrical case (1947) used a combination of linear velocity distribution and uniform density in the expression of kinetic energy, with a simplified, non-rigorous treatment of the explosive energy, yielding the formula

$$V_0 = \sqrt{2E_0}\left\{\frac{M}{C} + \frac{1}{6}\frac{3 + b}{1 + b}\right\}^{-1/2} \tag{16}$$

Jones (1965) and Henry (1967) analyzed cylinders and spheres with rigid cores. Both used the classical Gurney approach combining the assumptions of uniform density and linear velocity distribution without considering bursting, and derived formulas that are quite simple: for cylindrical charges,

$$V_0 = \sqrt{2E_0}\left\{\frac{M}{C} + \frac{1}{6}\frac{3R_0 + R_c}{R_0 + R_c}\right\}^{-1/2} \tag{17}$$

and for spherical charges,

$$V_0 = \sqrt{2E_0}\left\{\frac{M}{C} + \frac{1}{10}\frac{6R_0^2 + 3R_0R_c + R_c^2}{R_0^2 + R_0R_c + R_c^2}\right\}^{-1/2} \tag{18}$$

Since these two formulas do not account for a bursting radius, they correspond to a late-time condition at which negligible internal energy remains in the explosive gases, while their velocity distribution remains linear; thus, these represent asymptotic expressions of the new Gurney formulas for linear velocity, Eqs. (12) and (13), as the bursting radius approaches infinity.

The above formulas for cylindrical charges, Eqs. (9), (12), (16), and (17), are compared in Fig. 2. The velocities predicted by the two new formulas fall 10% to 20% below the two previous formulas of Sterne and Jones; this is mainly because the new formulas fully account for a bursting radius, in both the remaining internal energy and the kinetic energy of the explosive products.

Also shown are the results of computations performed using the 1-D capability of the EPIC code (Johnson et al., 1994). These computations used the LX-14 explosive, modeled by the JWL equation of state, with an initial density of 1.838 g/cm^3, and a steel liner of density = 7.89 g/cm^3. The rigid core is handled by constraining the inner boundary of the explosive, and all of the explosive is instantaneously initiated. Velocities when the liner has expanded to 1.6 times its initial size (i.e., $r_0/R_0 = 1.6$) are plotted. These fall between the curves for the new formulas, in which the Gurney

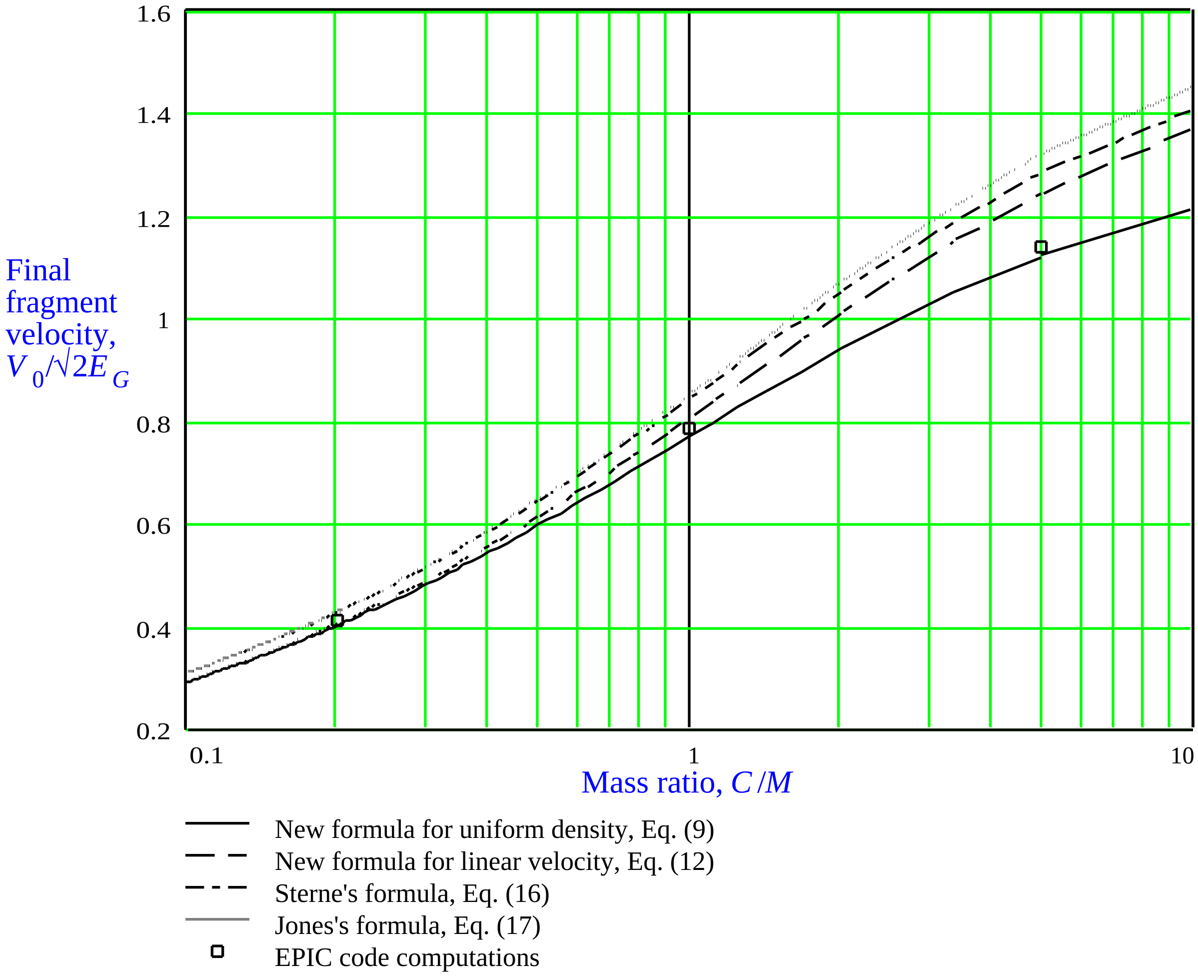

Fig. 2. Comparison of Gurney formulas and hydrocode computations for cylindrical charges with rigid cores, for a core-radius ratio $R_c/R_0 = 0.6$ and a bursting-radius ratio $r_b/R_0 = 1.6$.

energy $\sqrt{(2E_G)}$ is taken as 3.05 km/s, and a ratio of specific heats $\gamma = 2.825$ is used (Eq. (14) is integrated numerically.). Actually, by adjusting the Gurney energy slightly, these points can be made to fall directly on either of these curves.

The reason for this is suggested by Figs. 3 and 4, which show distributions of velocity and density in the explosive products at the time when the liner has expanded to 1.6 times its initial radius. The code predictions fit the curves corresponding to the two new models perhaps equally well but each with some difference. The velocity distributions in Fig. 3 are not quite linear but not as non-linear as the curve corresponding to uniform density. Likewise, in Fig. 4, the distributions of ρ are not uniform but neither are they non-uniform in a manner of a linear velocity distribution.

The formulas for spherical charges, Eqs. (10), (13), (15), and (18), are compared in Fig. 5. Again, the velocities predicted by the two new formulas fall below the two previous formulas. The hydrocode-predicted velocities again agree best with the new formula based on uniform density, Eq. (10).

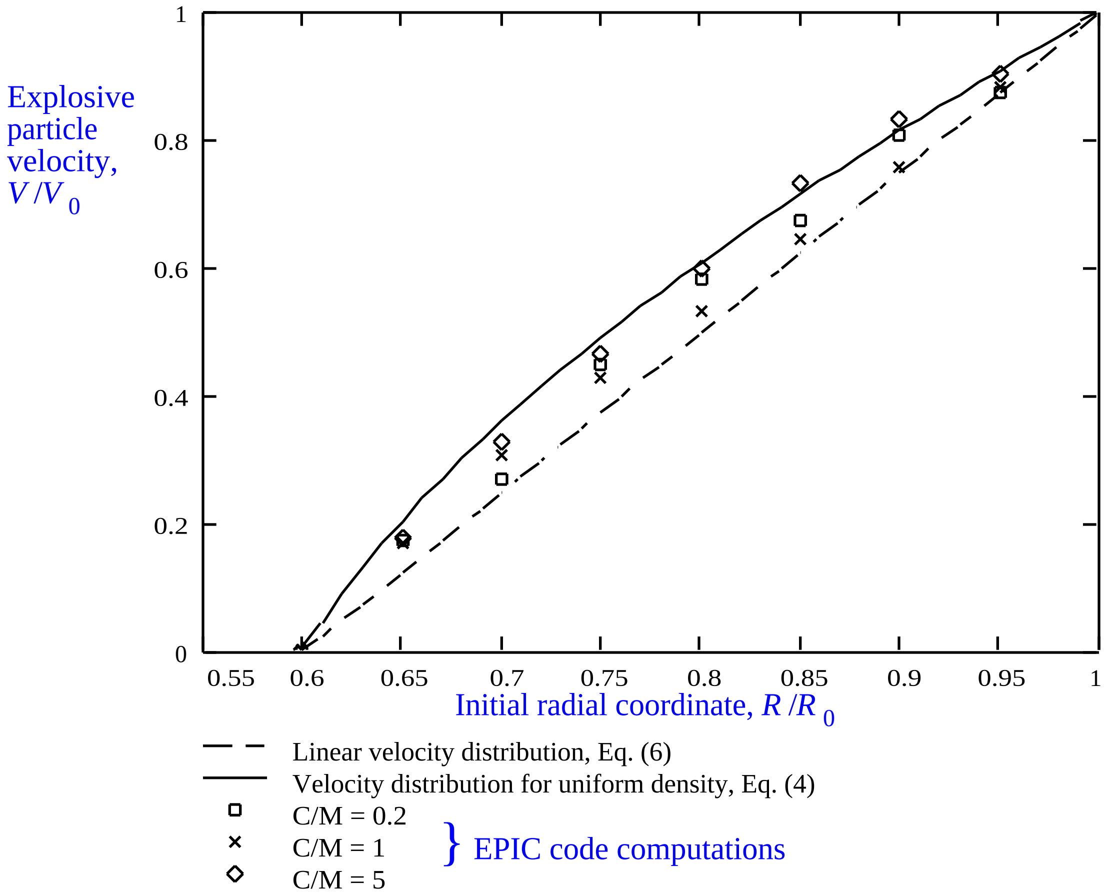

Fig. 3. Velocity distribution in the explosive products for a core-radius ratio $R_c/R_0 = 0.6$ and a bursting-radius ratio $r_b/R_0 = 1.6$.

EFFECT OF RIGID CORE

Hirsch (1986) observed that Jones's formulas, Eqs. (17) and (18), predict that adding a small rigid core increases the velocity of the liner, even at the expense of the displaced explosive. This behavior may be examined in any of the Gurney formulas mentioned here by replacing the explosive mass term C by the expression $C_0\,(1 - R_c^n/R_0^n)$, where C_0 represents the charge mass for the case of no core. Thus, Jones's formula for cylinders ($n = 2$), Eq. (17), becomes

$$V_0 = \sqrt{2E_0}\left\{\frac{M}{C_0\left[1-(R_c^2/R_0^2)\right]} + \frac{1}{6}\,\frac{3R_0+R_c}{R_0+R_c}\right\}^{-1/2}$$

which predicts a maximum liner velocity for a core of radius given by

$$\frac{R_c}{R_0} = 1 + 3\frac{M}{C_0} - \sqrt{\left(1+3\frac{M}{C_0}\right)^2 - 1}$$

Velocities predicted by our new formulas, modified in a similar manner, are plotted versus core radius in Fig. 6 for cylindrical charges. These curves show that the predicted increase in velocity due to adding a core is small, especially for smaller values of C_0/M. Also, the new formula based on uniform density, Eq. (9), predicts a negligible increase for $C_0/M = 5$ and none for the other cases.

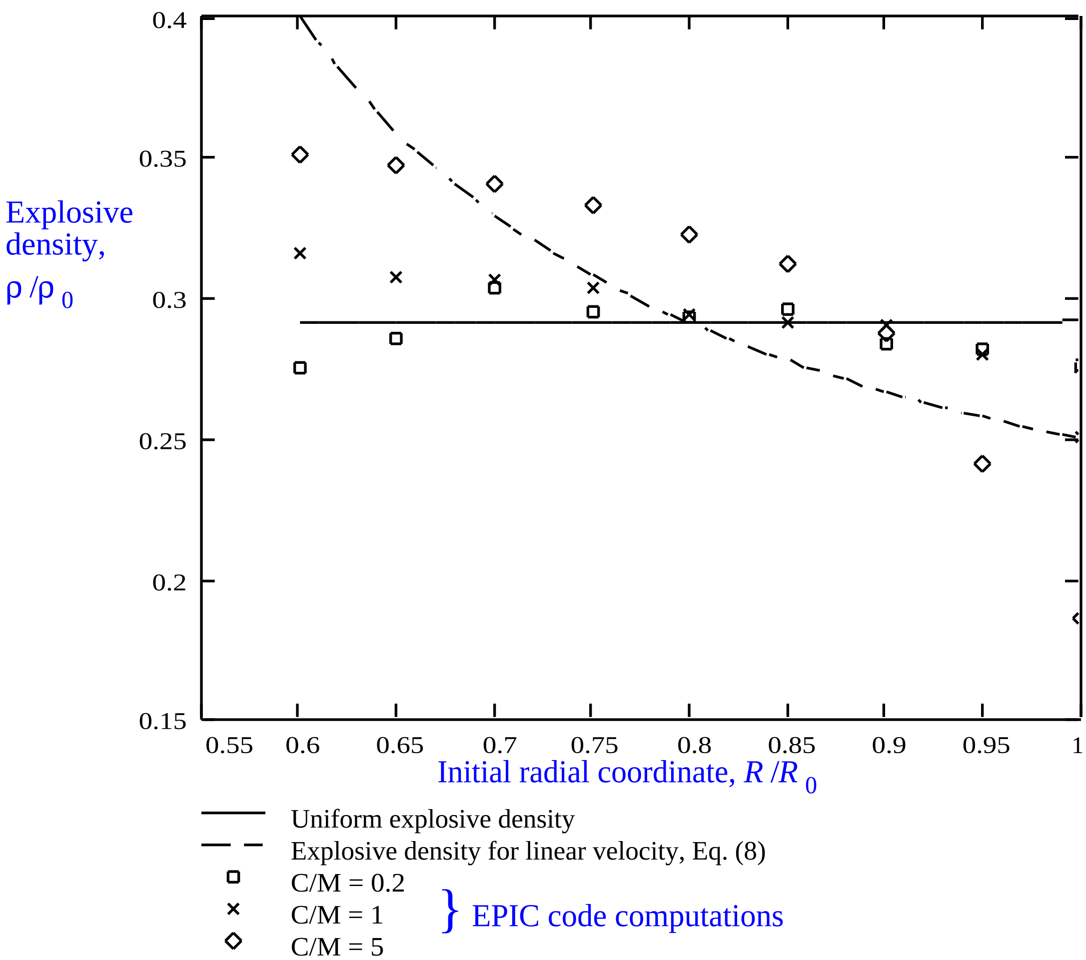

Fig. 4. Density distribution in the explosive products for a core-radius ratio $R_c / R_0 = 0.6$ and a bursting-radius ratio $r_b / R_0 = 1.6$.

This figure also includes the results of 1-D EPIC code computations, which generally agree well with the predictions of the formulas, and, like for the formula for uniform density, show a negligible increase in velocity with the addition of a small rigid core. The best agreement is with the formula based on the assumption of uniform density of the explosive products.

However, it is interesting that both the formulas and the code computations show that there is only a very small decrease in velocity when a core of significant size is added. In particular, for $C_0/M = 5$, the code-predicted velocities are remarkably flat, equal to five significant digits for cores having radii up to 60% that of the explosive charge. Even for a core with a radius equal to 80% that of the explosive charge, the code-predicted velocity is only 4% less than for no core, even though the explosive mass is 64% less. For lower C_0/M values, the code-predicted velocities are less flat, initially increasing slightly with core radius, then decreasing more quickly.

A similar study for spheres is shown in Fig. 7. Both the curves and the code computations of velocity are even flatter with respect to core radius than for cylinders; this is expected because the fraction of explosive mass displaced by the core is equal now to the cube of the core-radius ratio R_c/R_0, rather than the square for cylinders.

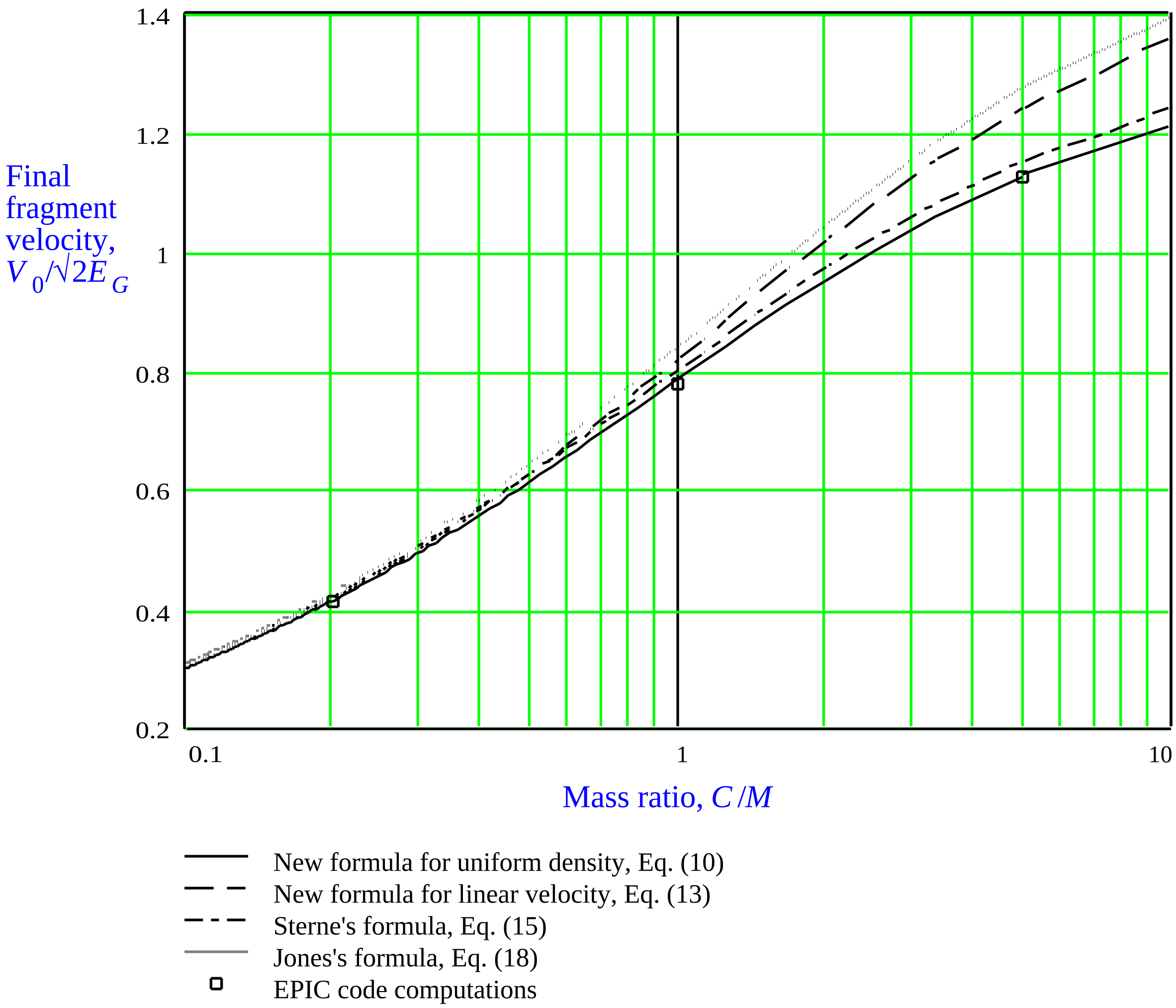

Fig. 5. Comparison of Gurney formulas and hydrocode computations for spherical charges with rigid cores, for a core-radius ratio $R_c/R_0 = 0.6$ and a bursting-radius ratio $r_b/R_0 = 1.6$.

CONCLUSION

The usual assumptions for deriving Gurney formulas, uniform density and linear velocity distribution in the explosive products, have been shown to be mutually inconsistent for cylindrical and spherical charges surrounding rigid cores. Theoretically consistent formulas have been derived for such charges by rigorously applying each of these assumptions individually; these also account for a finite bursting radius of the liner. Comparisons with previous formulas show some differences, but comparison with hydrocode computations shows that neither assumption is exactly valid. An interesting prediction of these formulas, verified by hydrocode, is that replacing some of the explosive by a core as large as 80% the explosive radius, results in a negligible decrease in liner velocity.

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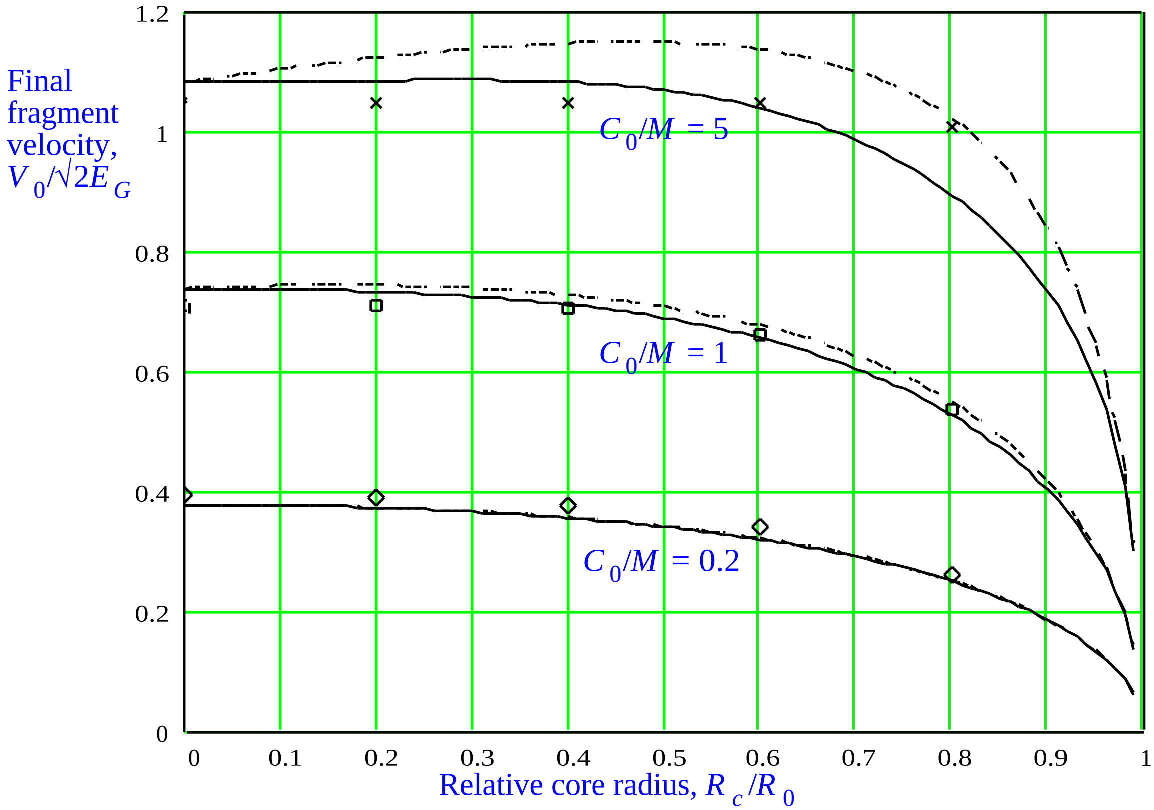

Fig. 6. Comparison of liner velocities of cored cylindrical charges, predicted by the new formulas, Eq. (9) (solid lines) and Eq. (12) (dashed lines), and hydrocode computations (symbols) for bursting-radius ratio $r_b/R_0 = 1.6$.

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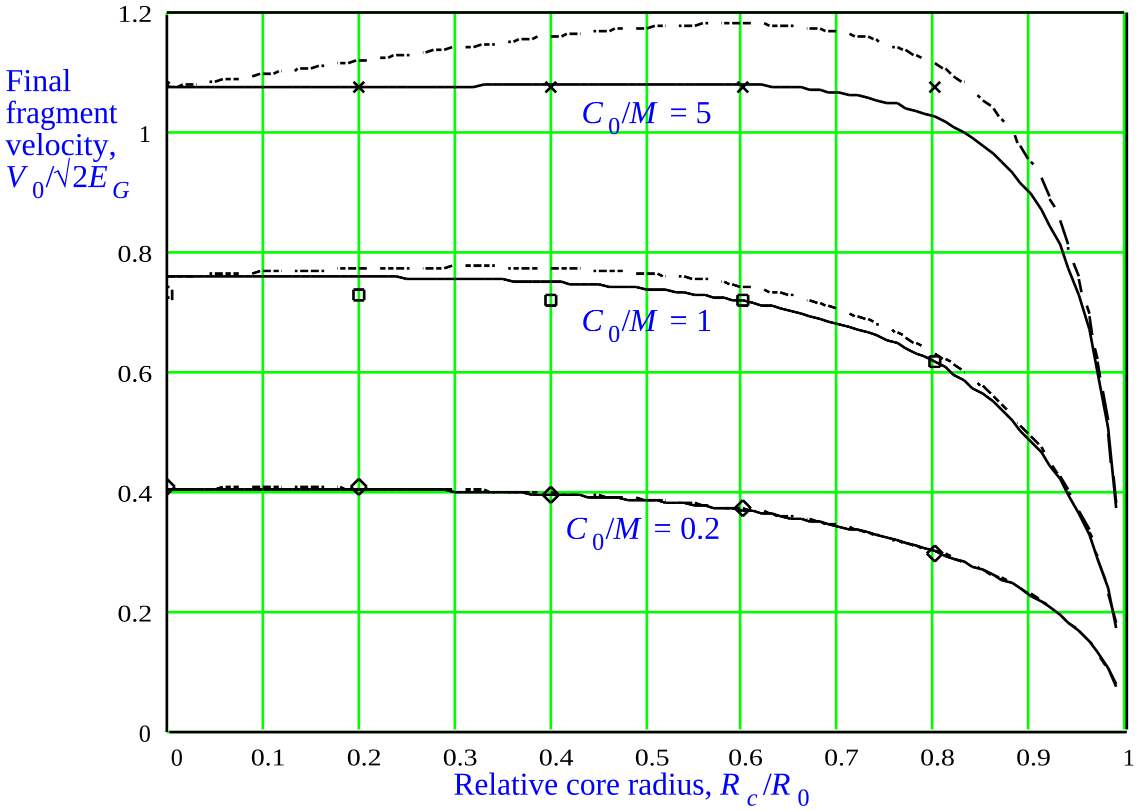

Fig. 7. Comparison of liner velocities of cored spherical charges, predicted by the new formulas, Eq. (10) (solid lines) and Eq. (13) (dashed lines), and hydrocode computations (symbols) for bursting-radius ratio r_b/R_0 = 1.6.